






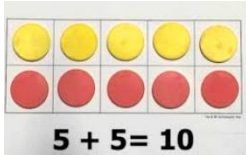
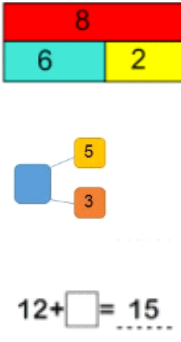


# HARROWBARROW SCHOOL



**Calculation Policy – February 2018**

EYFS	Concrete	Pictorial
<p>Children count reliably with numbers from 1 -20.</p>	<ul style="list-style-type: none"> <li>• Number songs with props.</li> <li>• Counting loose objects and developing 1-1 correspondence.</li> </ul> 	<ul style="list-style-type: none"> <li>• Daily 100 day counting using 100 square.</li> <li>• Counting across number line.</li> </ul>
<p>Children recognise and place numbers from 1 – 20 in order.</p>	<ul style="list-style-type: none"> <li>• Play based learning using resources clearly labelled with numerals to support recognition;</li> </ul> 	<ul style="list-style-type: none"> <li>• Ordering items either by quantity or numeral.</li> </ul> 
<p>Say which number is 1 more or 1 less than a given number.</p>	<ul style="list-style-type: none"> <li>• Acting out number songs, e.g. 5 Little Speckled Frogs; removing 1 frog at a time.</li> </ul> 	<ul style="list-style-type: none"> <li>• Using bead strings, numicon, unifix, etc to add/subtract 1.</li> <li>• Using a number line to visualise where 1 more/less is.</li> </ul>
<p>Using quantities and objects, they add and subtract 2 single-digit numbers and count on or</p>	<ul style="list-style-type: none"> <li>• Using loose parts to create number sentences, e.g. throwing coloured on one side beans onto a plate, how many gold?</li> </ul>	<ul style="list-style-type: none"> <li>• Using numicon, bead strings, loose parts, etc. to generate</li> </ul>

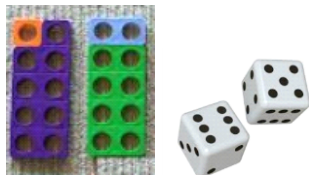
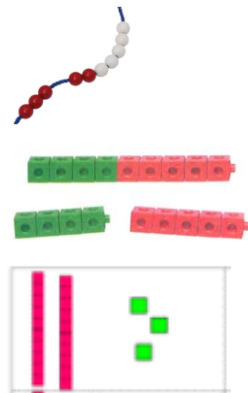
<p>back to find the answer.</p>	<p>How many white?</p>  <ul style="list-style-type: none"> <li>Practical contexts, if there are 4 people in the house and this child joins how many will there be?</li> </ul>	 <p>number sentences.</p> <ul style="list-style-type: none"> <li>Rolling dice and adding/subtracting quantities.</li> </ul>	
<p>They solve problems including doubling, halving and sharing.</p>	<ul style="list-style-type: none"> <li>Discussing doubles when playing dice games/dominos. Have you scored a double?</li> <li>Using physical objects to create doubles; I have 4 aliens, how many would I have if I added another 4?</li> <li>Cutting cakes, fruit, play dough, etc. into halves.</li> </ul>  <ul style="list-style-type: none"> <li>Sharing items equally between 2 children.</li> <li>Sharing items equally between a small group of children.</li> </ul>	<ul style="list-style-type: none"> <li>Pictures of collections of objects. Can you add another set of the same quantity, how many altogether?</li> <li>Pictures of collections of objects. Can you share these between 2/3/4?</li> </ul>	
<p><b>Addition</b></p>	<p>Concrete</p>	<p>Pictorial</p>	<p>Abstract</p>
<p><b>Year 1</b> Children should read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs represent and use number bonds and related subtraction facts within 20 add and subtract one-digit and</p>			

two-digit numbers to 20, including zero  
Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = ? - 9$ .

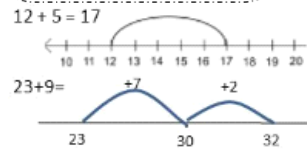
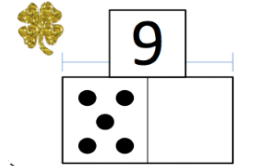
**Concrete** methods might include:  
using beadstrings, Numicon, multilink or unifix to make number bonds to 20, using dot patterns on dice to add, using Numicon to balance a simple equation such as  $7 = 3 + 4$

Pupils will then move on to more **pictorial** representations such as using 10 frames, part-part whole with missing numbers or numbers represented by pictures, using numberlines to count on.

**Abstract** maths will involve being able to put missing numbers in a box, looking at patterns when adding numbers such as  $5 = 4 + 1$ ,  $5 = 2 + 3$  etc



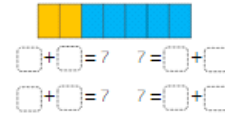
#4 What's the missing part?



$$4 + 12 =$$

$$= 4 + 15$$

2 Complete the number sentences.



$$\text{blue circle} + \text{red triangle} = 4$$

$$\text{red triangle} + \text{blue circle} = 4$$

$$4 = \text{red triangle} + \text{blue circle}$$

$$4 = \text{blue circle} + \text{red triangle}$$

$$\text{5 red circles} = 5 = 5 + 0$$

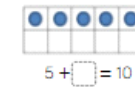
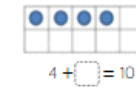
$$\text{4 red circles and 1 yellow circle} = 4 = 4 + 1$$

$$\text{3 red circles and 2 yellow circles} = \dots = \dots + \dots$$

$$\text{2 red circles and 3 yellow circles} = \dots = \dots + \dots$$

$$\text{1 red circle and 4 yellow circles} = \dots = \dots + \dots$$

$$\text{0 red circles and 5 yellow circles} = \dots = \dots + \dots$$



**Year 2**

Children should solve one-step problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; adding three one-digit numbers show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

**Concrete** maths might involve:

Concrete objects and pictorial representations for addition and subtraction. Includes ten frames with vertical bars and small squares, and dot patterns.

$8 + 6$  leads to  $8 + 2 + 4$

46

Vertical bar and square representations for addition and subtraction problems.

Money examples: A ten frame with four coins, and a problem asking "How much money is shown in total?" with various coins (50p, 20p, 20p, 5p, 2p).

Top Notch Teaching logo.

$7 + 5$  using ten frames with frog icons.

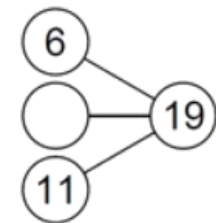
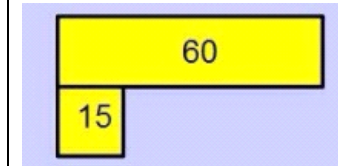
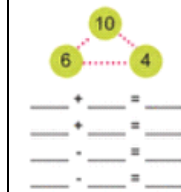
$7 + 3 + 2$  using ten frames with frog icons.

Three black pentagons.

$5 + 5 + 5$

10 less		10 more
	12	22
	37	

$$\begin{array}{r} 38 + 27 \\ \begin{array}{r} 30 \quad 8 \\ 20 \quad 7 \end{array} \end{array}$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

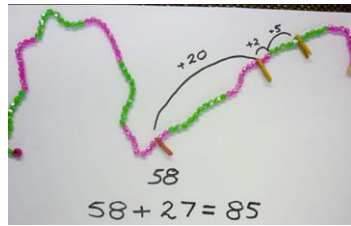
$25 + 37 = 62$

bridging the gap with Numicon (understanding that  $8 + 6$  could be worked out as  $8 + 2 + 4$ ), partitioning numbers using Dienes, using Cuisenaire rods to work out addition questions such as  $8 + 5 = 10 + 3$ , using beadstrings to count on tens and then units

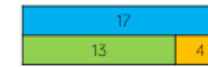
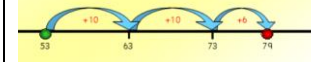
**Pictorial** maths might include: using ten frames to work out addition sums such as  $7 + 5 = 7 + 3 + 2$ , working out sums of money, using shape to work out addition questions, using Dienes to start to partition numbers

**Abstract** maths might include: working out part/part whole models or bar models, using a 100 square to count on tens and units, using a numberline to count on tens and units, using knowledge of  $3 + 7$  to work out  $30 + 70$ , starting to use column methods for addition

$$8+5$$



$$53 + 26 = 79$$



- |          |                       |          |
|----------|-----------------------|----------|
| $6 + 4$  | <input type="radio"/> | $6 + 5$  |
| $6 + 4$  | <input type="radio"/> | $3 + 6$  |
| $11 - 4$ | <input type="radio"/> | $12 - 5$ |
| $11 - 4$ | <input type="radio"/> | $12 - 4$ |

Tens	Ones
	· ·

$$\begin{array}{r} 23 \\ + 40 \\ \hline \end{array}$$

2 Complete the part whole models below:



3 Find the missing numbers in the related facts.

$5 + 4 = 9$	$8 = 3 + 5$	$4 = 10 - 6$
$50 + 40 = \square$	$80 = 30 + \square$	$40 = \square - 60$

### Year 3

Children should:

add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds

add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

estimate the answer to a calculation and use inverse operations to check answers.

solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)

**Concrete** maths might include: using counters on place value charts, using Dienes to add, using Numicon to add tens and units

**Pictorial** maths might include: using place value charts to support the idea of column addition, finding missing

Concrete resources include a place value chart with ten blocks, Numicon pieces for 100, 10, and 1, and base ten blocks (tens rods and ones units).

1 Solve  $245 + 7$

*I can add two and three digit numbers*

Hundreds	Tens	One
7	7	7

Part of the place value chart is covered. Can you work out the calculation?

*I can add two and three digit numbers*

Hundreds	Tens	Ones
5	3	3
	4	2

(a)

5	1	3
-	2	1
3	1	5

**Addition: Column Method**

- Place the numbers one on top of the other, lining up the tens and ones.
- Add the ones and write the answer under the ones.
- Regroup any tens to the tens column.
- Add the tens including any one you've regrouped.
- Check your answer.

Use the bar model to complete the number sentences.

600	_____ = 600	600 = _____ + _____
200	_____ = 600	600 = _____ + _____
400	_____ = 400	400 = _____ + _____
	_____ = 200	200 = _____ + _____

2 Calculate three hundred and seventy nine add five.

3 Use a number line to calculate  $346 + 7$

46 + 7 = 53  
300 + 55 = 355

Complete the bar model.

176	40
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2 Use column addition to work out:

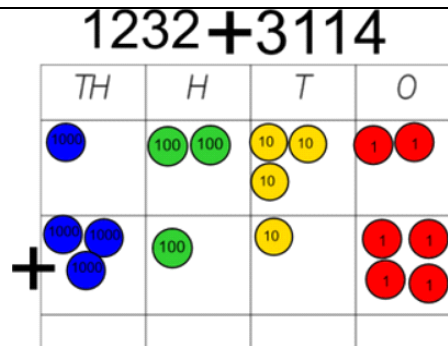
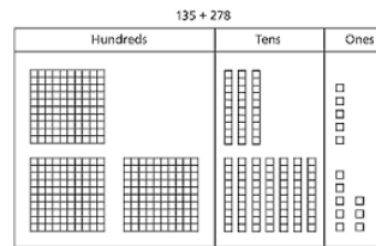
29	367
254	68

numbers using place value charts, solving problems using images of Dienes

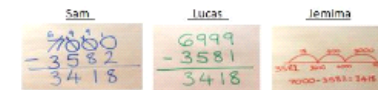
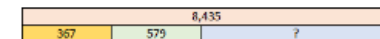
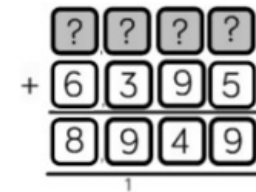
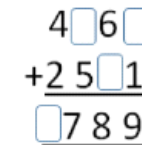
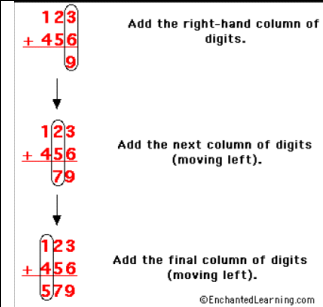
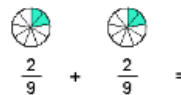
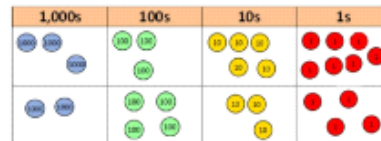
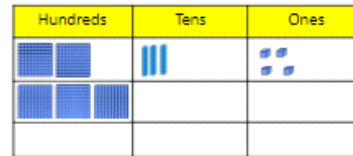
**Abstract** maths might include: part/part whole models, bar models, finding related facts for addition and subtraction, using column addition

**Year 4**

Children should add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why add and subtract fractions with the same denominator solve simple measure and money problems involving fractions and decimals to two decimal places. solve problems involving



Use the place value grid to help you work out two hundred and thirty-four add three hundred.





converting from hours to minutes; minutes to seconds; years to months; weeks to days.

**Concrete** maths might include: using counters on place value charts, using Dienes to add up to 4 digits

**Pictorial** maths might include: using place value charts to support the idea of column addition, finding missing numbers using place value charts, solving problems using images of Dienes, using fraction chart to add fractions with the same denominator

**Abstract** maths might include: part/part whole models, bar models, finding related facts for addition and subtraction, using column addition, link to measure such as working out perimeter of shapes



$$200 + 40 + 7$$

$$100 + 20 + 5$$

$$300 + 60 + 12 = 372$$

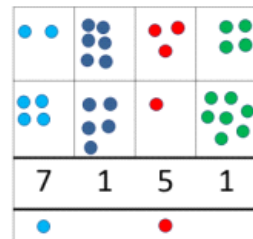
$$\begin{array}{r} 247 \\ +125 \\ \hline 12 \\ 60 \\ 300 \\ \hline 372 \end{array}$$

**Year 5**

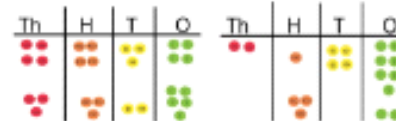
Children should add and subtract whole numbers with more than 4 digits, including using efficient written methods (columnar addition and subtraction) add and subtract numbers mentally with increasingly large numbers

**Compact written method**

Extend to numbers with at least four digits



2  
+4  
7



$$\begin{array}{r} 430 \\ + 252 \\ \hline 7852 \end{array}$$

use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

add and subtract fractions with the same denominator and related fractions; write mathematical statements  $>1$  as a mixed number (e.g.  $2/5 + 4/5 = 6/5 = 11/5$ )

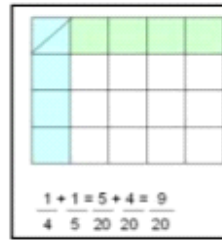
**Concrete** maths might include: using counters on place value charts, using Dienes to add up to 4 digits

**Pictorial** maths might include: using place value charts to support the idea of column addition including adding decimals, finding missing numbers using place value charts, solving problems using images of Dienes, using fraction chart to add fractions with denominators in the same times tables

**Abstract** maths might include: part/part whole models, bar models, finding related facts for addition and subtraction, using



Thousands			Ones			Decimals		
hundreds	tens	ones	hundreds	tens	ones	tenths	hundredths	thousandths
2	2	2	2	2	2	2	2	2



$$\begin{array}{r} \pounds 563.14 \\ + \pounds 207.88 \\ \hline \pounds 771.02 \\ \hline 111 \end{array}$$

$$\begin{aligned} 12\,462 + 2\,300 \\ = 12\,462 + 2\,000 + 300 \\ = 14\,462 + 300 \\ = 14\,762 \end{aligned}$$

$$\begin{aligned} 234\text{ kg} + 49\text{ kg} &= 273\text{ kg} \\ 200 + 30 + 4 \\ \quad \quad 40 + 9 \\ \hline 200 + 70 + 13 \end{aligned}$$

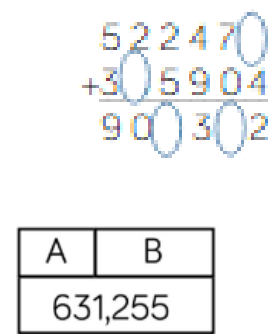
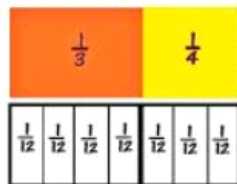
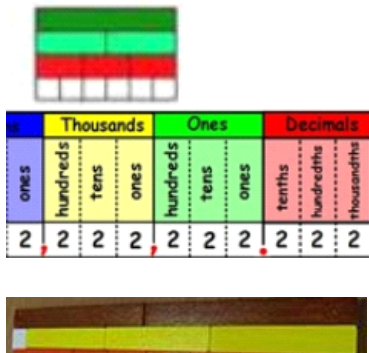
column addition

They should be encouraged to choose from a range of strategies:

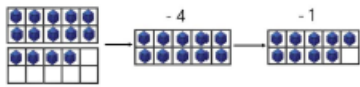





- Counting forwards and backwards in tenths and hundredths:  $1.7 + 0.55$
- Reordering:  $4.7 + 5.6 - 0.7$ ,  $4.7 - 0.7 + 5.6 = 4 + 5.6$
- Partitioning: counting on or back -  $540 + 280$ ,  $540 + 200 + 80$
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating:  $5.7 + 3.9$ ,  $5.7 + 4.0 - 0.1$
- Partitioning: using 'near' double:  $2.5 + 2.6$  is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.

**Year 6**

Children should use their knowledge of the order of operations to carry out calculations involving the four operations use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. solve addition and subtraction multi-step problems in contexts, deciding which operations and



<p>methods to use and why add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions</p> <p><b>Concrete</b> maths might include: using counters on place value charts, using Dienes to add</p> <p><b>Pictorial</b> maths might include: using place value charts to support the idea of column addition including adding decimals to 3 dp, finding missing numbers using place value charts, solving problems using images of Dienes, using fraction chart to add fractions</p> <p><b>Abstract</b> maths might include: part/part whole models, bar models, finding related facts for addition and subtraction, using column addition</p>			<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>Line up the decimal points</p> <p>↓</p> <math display="block">\begin{array}{r} 22.3 \\ + 34.1 \\ \hline 56.4 \end{array}</math> </div> <div style="text-align: center;"> <p>Line up the decimal points</p> <p>↓</p> <math display="block">\begin{array}{r} 1.234 \\ + 4.1 \\ \hline 5.334 \end{array}</math> </div> </div>
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Subtraction	Concrete	Pictorial	Abstract
<p><b>Year 1</b></p> <p>Pupils should:</p> <p>read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs</p> <p>represent and use number bonds and related subtraction facts within 20</p> <p>add and subtract one-digit and two-digit numbers to 20, including zero</p>	<p>14 - 5</p>  <p>- 4                      - 1</p> 	<p>Complete the number sentence</p>  <p><math>7 - 2 = \square</math></p>  <p><math>6 - 2 = \square</math></p> 	<p><math>14 - 5 = 9</math></p>  <p><math>14 - 4 = 10</math> <math>10 - 1 = 9</math></p>

Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = ? - 9$

**Concrete** methods might include:  
 using beadstrings, Numicon, multilink or unifix to make number bonds to 20, using dot patterns on dice to add, using Numicon to balance a simple equation such as  $7 = 10 - 3$   
 Pupils will then move on to more **pictorial** representations such as using 10 frames, part-whole with missing numbers or numbers represented by pictures, using numberlines to count back.  
**Abstract** maths will involve being able to put missing numbers in a box, looking at patterns when subtracting numbers such as  $5 = 6 - 1$ ,  $5 = 7 - 2$  etc

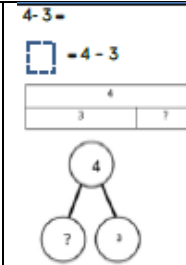
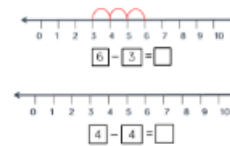
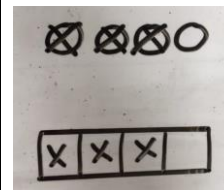


Think of two questions to ask your friend about the image.



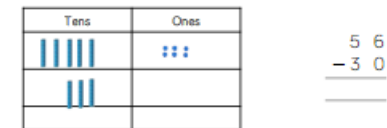
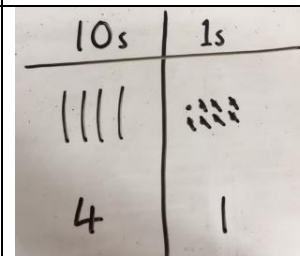
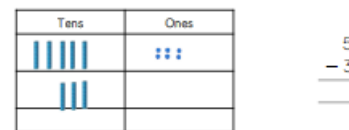
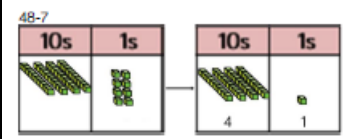
Represent them about the calculation.

$$\square - \square = \square$$



**Year 2**

Children should solve one-step problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods  
 recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

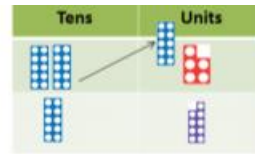
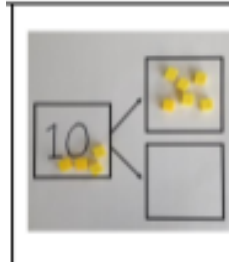


add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; adding three one-digit numbers show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

**Concrete** maths might involve: bridging the gap with Numicon (understanding that  $12 - 6$  could be worked out as  $12 - 2 - 4$ ), partitioning numbers using Dienes, using Cuisenaire rods to work out addition questions such as  $10 - 4$ , using beadstrings to count on tens and then units, using multilink to find the difference between two sets of blocks, using part/part whole models to link to addition

**Pictorial** maths might include: using ten frames to work out addition sums such as  $10 - 3 = 9 - 2$ , working out sums of money, counting back using a number starting using the larger number and counting back in ones and then tens, using Dienes to start to partition numbers, drawing bars to find the difference between two numbers, use base ten against written calculation to show how you could work this out

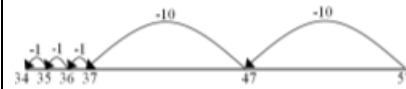
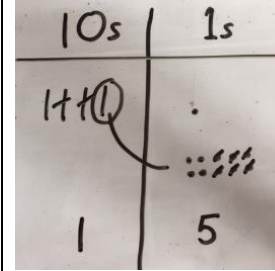
**Abstract** maths might include: working out part/part whole models or bar models, using a 100 square to count on tens and units, using a



Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.

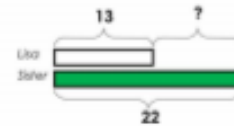


$$82 - 50 =$$

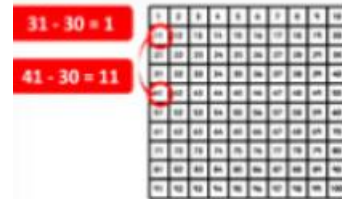
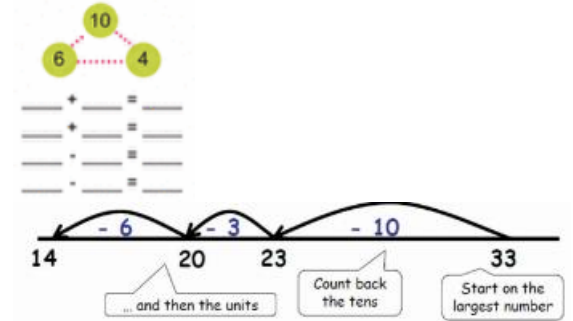


### Comparison Bar Models

Lita is 73 years old. Her sister is 22 years old. Find the difference in age between them.



$$73 - 40 = 33$$



numberline to count on tens and units, using knowledge of  $10 - 3 = 7$  to work out  $100 - 30$ , starting to use column methods for subtraction

**Year 3**

Children should:

add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds

add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

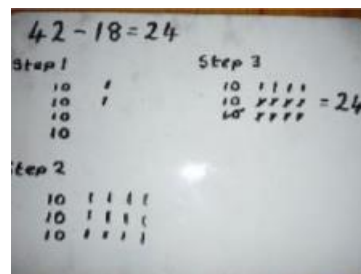
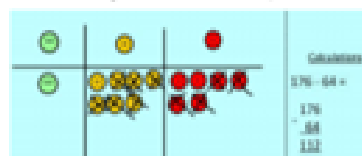
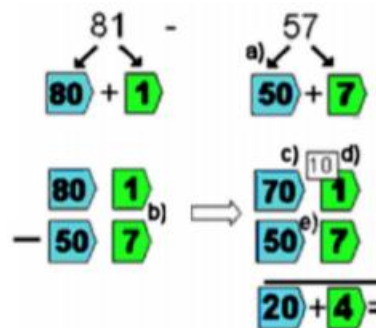
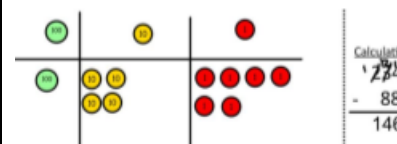
estimate the answer to a calculation and use inverse operations to check answers.

solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

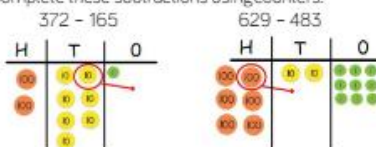
measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)

**Concrete** maths might include: using counters on place value charts, using Dienes to add, using Numicon to add tens and units, linking counters on place value chart to column methods for subtraction

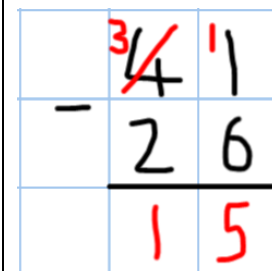
**Pictorial** maths might include: using place value charts to support the idea of column subtraction, finding missing numbers using place value charts, solving problems using images of Dienes, using jottings to exchange and regroup, use bar models to work out number



Complete these subtractions using counters.



$564 - \square = 558$   
 $\square - 8 = 725$   
 $352 = 361 - \square$



sentences  
**Abstract** maths might include: part/part whole models, bar models, finding related facts for addition and subtraction, using column addition, using bar model link to addition

45  
~~29~~  
 16

Tens | Ones

10 + 6 = 16

Children may draw base ten or PV counters and cross off.

836 - 254 = 582

Begin by partitioning into pv columns

728 - 582 = 146

Then move to formal method.

**Year 4**  
 Children should  
 add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why  
 solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why  
 add and subtract fractions with the same denominator  
 solve simple measure and money problems involving fractions and decimals to two decimal places.  
 solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.

234 - 179

Subtract 2,352 from the number below.

Hundreds	Tens	Ones

$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

Complete this subtraction problem.

Thousands	Hundreds	Tens	Ones
7	6	4	6
- 4	3	3	5

Find the missing numbers that could go into the boxes.

Give reasons for your answers.

- 1,345 = 4  6

8,435		
367	579	?

3465	
2980	

Look at each pair of calculations below. Which one out of each pair of calculations has the same difference as 2450 - 1830?

2,451 - 1,831 =      2,451 - 1,829 =  
 2,500 - 1,880 =      2,500 - 1,780 =  
 2,449 - 1,829 =      2,449 - 1,831 =

**Concrete** maths might include: using counters on place value charts, using Dienes to add, using



Numicon to add tens and units, linking counters on place value chart to column methods for subtraction

**Pictorial** maths might include: using place value charts to support the idea of column subtraction, finding missing numbers using place value charts, solving problems using images of Dienes, using jottings to exchange and regroup, use bar models to work out number sentences

**Abstract** maths might include: part/part whole models, bar models, finding related facts for addition and subtraction, using column addition, using bar model link to addition, working out which sums have the same answer using their understanding of place value

$$\begin{array}{r} 3.80 \leftarrow \\ - 1.26 \\ \hline \end{array}$$

Using zero as a place holder to help with regrouping

## Year 5

Children should

add and subtract whole numbers with more than 4 digits, including using efficient written methods (columnar addition and subtraction)  
add and subtract numbers mentally with increasingly large numbers

use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

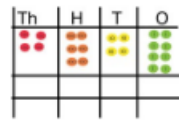
add and subtract fractions with the same denominator and related fractions; write mathematical statements  $>1$  as a mixed number (e.g.  $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$ )

**Concrete** maths might include: using counters on place value charts, using Dienes to subtract up to 4 digits

**Pictorial** maths might include: using place value charts to support the idea of column subtraction including adding decimals, finding missing numbers using place value charts, solving problems using images of Dienes, using fraction chart to subtract fractions with denominators in the same times tables

**Abstract** maths might include: part/part whole models, bar models, finding related facts for addition and subtraction, using column subtraction

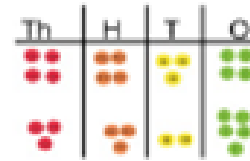
$$4,648 - 2,347$$



$$45,536 - 8,426$$



Thousands			Ones			Decimals		
hundreds	tens	ones	hundreds	tens	ones	tenths	hundredths	thousandths
2	2	2	2	2	2	2	2	2



$$\begin{array}{r} \cancel{8} \cancel{1} \cancel{0} \cancel{8} \cancel{6} \\ - \quad 2 \quad 1 \quad 2 \quad 8 \\ \hline 2 \quad 8 \quad 9 \quad 2 \quad 8 \end{array}$$

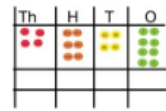
Use zeros for place-holders.

$$\begin{array}{r} \cancel{7} \cancel{1} \cancel{6} \cancel{8} \cdot \cancel{0} \\ - \quad 3 \quad 7 \quad 2 \cdot 5 \\ \hline 6 \quad 7 \quad 9 \quad 6 \cdot 5 \end{array}$$

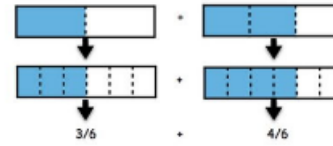
**Year 6**

Pupils should:  
use their knowledge of the order of operations to carry out calculations involving the four operations solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

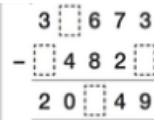
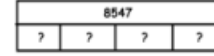
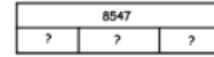
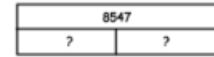
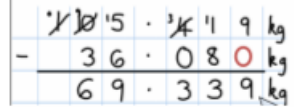
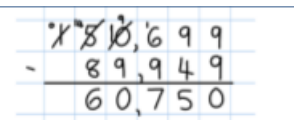
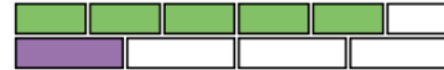
4,648 - 2,347



45,536 - 8,426



$\frac{5}{6} - \frac{1}{4} = \square$



# Multiplication

## Year 1

Pupils should be taught to: -step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

**Concrete:** counting sticks, 100 squares and numberlines, Numicon, using fingers for counting up, dominoes, dice, printing in paints, using handprints, counting in 2s with socks, arrays through Numicon as well as chocolate bars

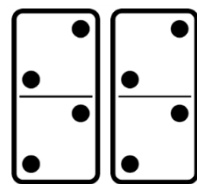
**Pictorial:** using arrays, grouping objects for counting, putting spots on a ladybird, drawing on blank dice,

**Abstract:** using numberlines, linking multiplication to repeated addition,

## Year 2

Pupils should be taught to: division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs numbers can be done in any order (commutative) and division of one number by another cannot

## Concrete

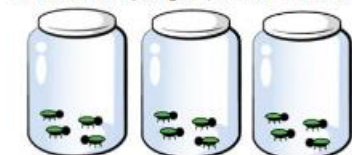


Repeated grouping/repeated addition

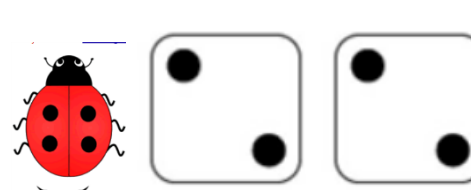
$$3 \times 4$$

$$4 + 4 + 4$$

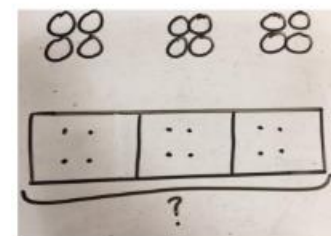
There are 3 equal groups, with 4 in each group.



## Pictorial



Children to represent the practical resources in a picture and use a bar model.



## Abstract

$$6 + 3 = 9$$

Repeated addition number sentence  
 $2 + 2 + 2$

Worded problem  lots of 2 is 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

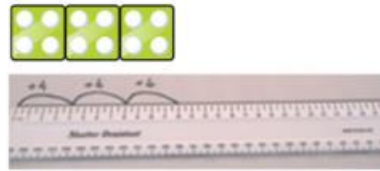
multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

**Concrete:** counting sticks, 100 squares and numberlines, Numicon for counting in steps of 2, 5 and 10, using fingers for counting up (Mexican wave), money counting up in 2, 5 and 10s, using different objects, Unifix, using shapes such as pentagons for counting up, dominoes, dice, counting in 2s with socks, arrays through Numicon as well as chocolate bars, packs of 10 pencils or cartons of drinks

**Pictorial:** using arrays, grouping objects for counting, putting spots on a ladybird, drawing on blank dice,

**Abstract:** using numberlines, linking multiplication to repeated addition,

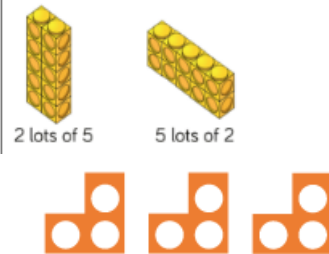
Number lines to show repeated groups-  
 $3 \times 4$



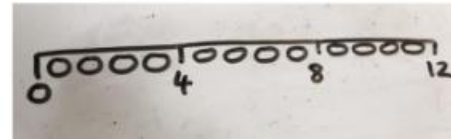
Cuisenaire rods can be used too.

Use arrays to illustrate commutativity counters and objects can also be used.

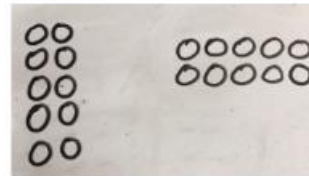
$2 \times 5 = 5 \times 2$



Represent this pictorially alongside a number line e.g:



Children to represent the arrays pictorially.

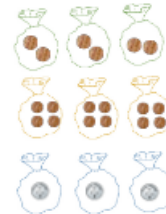


Complete the stem sentence.



There are \_\_\_ equal groups with \_\_\_ in each group.

Which group of money is the odd one out?



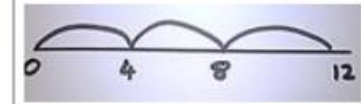
Explain why.

How can you make the groups equal?



Abstract number line showing three jumps of four.

$3 \times 4 = 12$



Children to be able to use an array to write a range of calculations e.g.

$10 = 2 \times 5$   
 $5 \times 2 = 10$   
 $2 + 2 + 2 + 2 + 2 = 10$   
 $10 = 5 + 5$

Complete the sentences to describe the equal groups.



There are \_\_\_ equal groups with \_\_\_ in each group.  
 There are three \_\_\_.

Use  $<$ ,  $>$  or  $=$  to make the statements correct.

$3 \times 5$          $5 + 5 + 5 + 5$   
 $2 \times 2$          $2 + 2$   
 $4 + 4 + 4$          $2 \times 2$

Sort into equal and unequal groups.

Equal groups | Unequal groups



Create your own picture to go in each column.

Which one does not belong?



Two 5s

Ten



$$5 + 5 =$$

What do we need to change to make them all represent the same?



$$\square \times \square =$$

$$\square \text{ lots of } 3 = \square$$

$$\square \text{ multiplied by } \square = 12$$

Part of the array is hidden.



The total is less than 16

What could the array be?

2	4	8	12
14	16	18	24
38	40	42	44

20	21	22	23	24	25	26	27	28
10	11	12	13	14	15	16	17	18
0	1	2	3	4	5	6	7	8

Complete the table:

Three 2s	Draw It	Addition	Multiplication
There are 3 equal groups with 2 in each group.			

Complete:

Addition	Multiplication	Story
10 + 10 + 10		
	6 × 5	

### Year 3

Children should:

recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

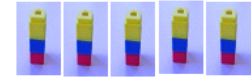
write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods  
solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects

#### Concrete methods:

linking multiplication to repeated addition, using cones, Numicon, multilink or Cuisenaire rods to look at arrays with related multiplication and division facts, using place value charts to multiply tens and units, using Numicon to group tens and units

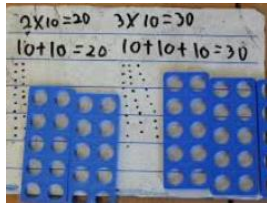
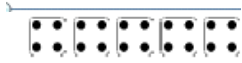
#### Pictorial methods:

Linking Numicon to more formal methods for multiplication such as the grid method, using shape to complete number statements, showing arrays pictorially, showing Dienes pictorially by multiplying out tens and units

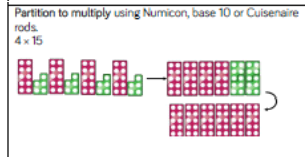
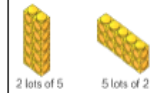


$$\_ + \_ + \_ + \_ + \_ =$$

$$\_ \times \_ =$$



Use arrays to illustrate commutativity counters and other objects can also be used.  
 $2 \times 5 = 5 \times 2$



Formal column method with place value counters (base 10 can also be used.)  $3 \times 23$

10s	1s
6	9

### multiply 1 and 2 digit numbers

Use your Numicon to work out:

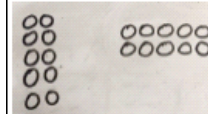
$2 \times 12$	$3 \times 13$



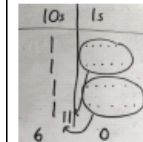
Complete the number sentences.

1 triangle has 3 sides.  $1 \times 3 = 3$   
3 triangles have \_\_\_ sides.  $\_ \times \_ = \_$   
\_\_\_ triangles have 6 sides.  $\_ \times \_ = 6$   
\_\_\_ triangles have 15 sides.  $\_ \times \_ = 15$

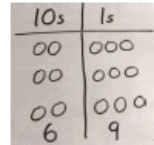
Children to represent the arrays pictorially.



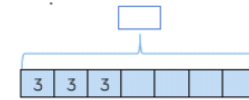
Children to represent the concrete manipulatives pictorially.



Children to represent the counters pictorially.



There are 7 tricycles in the playground.  
How many wheels are there altogether?  
Complete the bar model to find the answer.



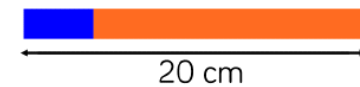
Here is a blue strip of paper.



An orange strip is four times as long.



The strips are joined end to end.



Children to be able to use an array to write a range of calculations e.g.

$10 = 2 \times 5$   
 $5 \times 2 = 10$   
 $2 + 2 + 2 + 2 + 2 = 10$   
 $10 = 5 + 5$

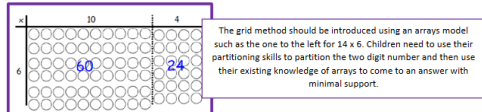
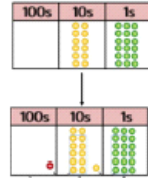
**Abstract:**

Completing bar models, using jottings to support concrete or pictorial methods by multiplying tens and units, starting to use formal methods for multiplication, linking multiplication to measure and problem solving

**Year 4**

Pupils should:  
 recall multiplication and division facts for multiplication tables up to  $12 \times 12$   
 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers  
 recognise and use factor pairs and commutativity in mental calculations  
 multiply two - digit and three - digit numbers by a one - digit number using formal written layout

Formal column method with place value counters.  $6 \times 23$

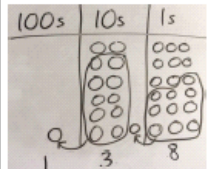


The grid method should be introduced using an arrays model such as the one to the left for  $14 \times 6$ . Children need to use their partitioning skills to partition the two digit number and then use their existing knowledge of arrays to come to an answer with minimal support.

x	30	5
7	210	35
	<b>210 + 35 = 245</b>	

Multiplication grid method requires good organization but also a solid understanding of partitioning and multiplication facts, as you can see in the example to the right for  $35 \times 7$ . The children need to remember that once they have multiplied the partitioned parts of the number, they then need to add the two

Children to represent the counters/base 10, pictorially e.g. the image below.



x	600	10	3
5	3000	50	15
	<b>613 x 5 = 3065</b>		

The compact 'short multiplication' method is tricky and needs to be approached carefully. At first children should solve a problem using grid method and then observe the teacher solve a problem using short multiplication and make comparisons. How are they similar? Children need to go through it very slowly and carefully, unpicking each step until they are fully confident.

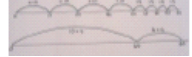
The grid method is extended in year 4 so children will now multiply 3 digit numbers by 1 digit numbers. When adding the 3 answers up to create a total, column addition could be used to ensure accuracy, especially where bridging will be needed.

	4	6	3
x	3	7	0
	12	42	0
	120	210	0
	1320	2520	0
	1320	2520	0

Children to be encouraged to show the steps they have taken.

$$\begin{array}{r} 4 \times 15 \\ 10 \times 3 \\ 10 \times 4 \\ 3 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$$

A number line can also be used



Children to record what it is they are doing to show understanding.

$$\begin{array}{r} 3 \times 23 \\ 20 \quad 3 \end{array} \quad \begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

$$6 \times 23 = 138$$

You have 10 paving stones to design a patio. The stones are one metre square. The stones must be joined to each other so that at least one edge is joined corner to corner.



Use squared paper to show which design would give the longest perimeter and which would give the shortest.



solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

**Concrete:** using counters on place value charts to work out x tens and units, using cones, multilink and Numicon to work out factors pairs for numbers


**Pictorial** Linking Numicon to more formal methods for multiplication such as the grid method, using shape to complete number statements, showing arrays pictorially, showing Dienes pictorially by multiplying out tens and units, use Dienes to multiply by 10

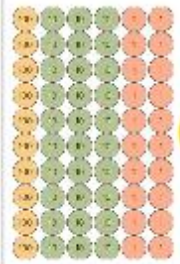

**Abstract:** link multiplication to measure questions such as area, converting units of measure and being able to x by 10 and 100, moving on to jottings and then more formal methods for multiplying 2 and 3 digit numbers by 1 digit

**Year 5**



Pupils should:  
 identify multiples and factors, including finding all factor pairs  
 solve problems involving multiplication and division where larger numbers are

Complete using <, > or =  
 Explain how you know.

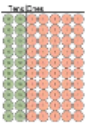
$221 \times 10$  

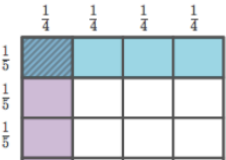
  $1,320 \times 10$  

Use Dienes to divide 140 by 10  
 Explain what you have done.

Write the calculation shown by the place value counters.

 Each row has \_\_\_ tens and \_\_\_ ones so each row has a value of \_\_\_  
 There are \_\_\_ rows.  
 The calculation is \_\_\_ x \_\_\_ = \_\_\_



$4 \times \square = 4$       $\square = 1 \times 7$   
 $0 = \square \times 42$       $63 \times 1 = \square$   
 $\square \times 27 = 0$       $50 \times \square = 50$

$3^3$	$3 \times 3 \times 3$	27
$5^3$	$5 \times 5 \times 5$	
	$6 \times 6 \times 6$	
$4^3$		8

used by decomposing them into their factors  
 know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers  
 establish whether a number up to 100 is prime and recall prime numbers up to 19  
 multiply numbers up to 4 digits by a one  
 -or two-digit number using an efficient written method, including long multiplication for two-digit numbers  
 recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)  
 solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign  
 solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates  
 multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

**Concrete:** using place value charts to support jottings for multiplying H T U, using arrays with Numicon or multilink for factors and to work out common factors, using multilink for working out



1323 x 3 =

1000	100	100	100	10	10	1	1	1
1000	100	100	100	10	10	1	1	1
1000	100	100	100	10	10	1	1	1

What's the same, what's different?

Th	H	T	1s	
1	3	2	3	
x			3	
	3	9	6	9

1324 x 3 =

What's stayed the same, what's changed?

1000	100	100	100	10	10	1	1	1
1000	100	100	100	10	10	1	1	1
1000	100	100	100	10	10	1	1	1
								100

What's stayed the same, what's changed?

Th	H	T	1s	
1	3	2	4	
x			3	
	3	9	7	2

Make the number 234 on the place value grid using counters.

HTh	TTh	Th	H	T	O
			●	●●	●●●

When I multiply my number by 10, where will I move my counters?  
 Remember when we multiply by 10, 100, 1000, we move the digits to the left and use zero as a place holder.

Use < > or = to complete the sentences.

62 x 1,000  62 x 100  
 100 x 32  32 x 100  
 48 x 100  48 x 10 x 10 x 10

Here are two methods to solve 24 x 20

Method 1	Method 2
24 x 10 x 2 = 240 x 2 = 480	24 x 2 x 10 = 48 x 10 = 480

What is the same about the methods, what is different?

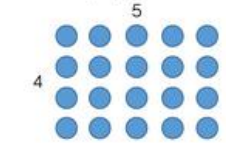
Write all the multiples of 4 between 20 and 80.

Children will use short multiplication in a range of increasingly challenging problems. Solving using the grid method and then comparing to the short multiplication method will help cement the children's understanding of the short multiplication method.

When multiplying by more than 1 digit, children need to use long multiplication. Like with short multiplication, they will solve the problem using the grid method first and then make comparisons until their understanding is secure. In the example below the top row shows 18 x 3 and the bottom shows 18 x 10. The final row shows the total of both calculations.

Once long multiplication methods are secure, children are ready to move on to more challenging problems which require greater levels of mental calculation. The problem to the right shows 1234 x 6 on the top line, 1234 x 10 on the bottom line and the total of both calculations on the final row.

If you have twenty counters, how many different ways of arranging them can you find? How many factors of twenty have you found? E.g. A pair of factors of 20 are 4 and 5.



squared and cubed numbers, using place value charts to work out  $\times$  decimals and whole numbers by 10, 100, 1,000 and 10,000

**Pictorial:** using images of place value charts or Dienes to support understanding of more formal methods of multiplication, using images such as overlays to work out multiplication of fractions

**Abstract:** using short and long multiplication methods being sure about place value, being able to work out multiplication of fractions, working out common factors, understand the multiples of a number

**Year 6**

multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g.  $1/4 \times 1/2 = 1/8$ )

multiply one digit numbers with up to two decimal places by whole numbers solve problems involving the calculations of percentages (e.g. of measures) such as 15% of 360 and the use of percentages for comparison

1323  $\times$  3 =

Th	H	T	1s	
1	3	2	3	
$\times$			3	
	3	9	6	9

What's the same, what's different?

1324  $\times$  3 =

Th	H	T	1s	
1	3	2	4	
$\times$			3	
	3	9	7	2

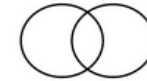
What's stayed the same, what's changed?

2 and  $2/3$   $\times$   $2/3$  =  $4$  and  $4/9$

$2/3 \times 4$

0  $2/3$  1  $4/3$  2  $8/3$  3 4

Fill in the Venn diagram to find the factors of 20 and 24.



Where are the common factors of 20 and 24? Can you use a Venn diagram to find the common factors of 9 and 15?

This table shows squared and cubed numbers. Complete the table. Explain the relationships you can see between the numbers.

	$3 \times 3$		$3^2$		27
		25	$5^2$		
$6^2$				$6 \times 6 \times 6$	
	$4 \times 4$		$4^2$		8
$9^2$					

96

32  $\times$

192  $\leftarrow$  this is  $96 \times 2$

2880  $\leftarrow$  this is  $96 \times 30$

3072  $\leftarrow$  this is  $96 \times 32$

3.19

$\times$  8

25.52

When multiplying decimals it is important to remember that the digit you are multiplying by needs to be lined up with the ones digits. As with all decimal work, the decimal points must be lined up and the children need to have a clear understanding why that is.

What are the common factors of these pairs of numbers?

- 24 and 36
- 20 and 30
- 28 and 45

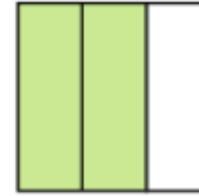
**Concrete:** using place value charts to support jottings for multiplying H T U, using arrays with Numicon or multilink for factors and to work out common factors, using multilink for working out squared and cubed numbers, using place value charts to work out  $\times$  decimals and whole numbers by 10, 100, 1,000 and 10,000, using Numicon to work out percentages

**Pictorial:** using images of place value charts or Dienes to support understanding of more formal methods of multiplication, using images such as overlays to work out multiplication of fractions, using Numicon to work out division of fractions by whole numbers

**Abstract:** using short and long multiplication methods being sure about place value, being able to work out multiplication of fractions, working out common factors, understand the multiples of a number, prime factors for a number, multiplying decimals, working out missing numbers for multiplication, working out what the largest number can be made through multiplication



First, separate the square into 3 equal parts vertically and shade 2 parts to indicate  $\frac{2}{3}$ .

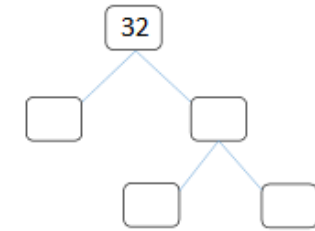


Now separate the square into 4 equal parts horizontally and shade 3 of them to show  $\frac{3}{4}$ .



Since we are looking for  $\frac{2}{3} \times \frac{3}{4}$  or  $\frac{2}{3}$  of  $\frac{3}{4}$ , we

get 6 out of 12 parts that are double-shaded and represent  $\frac{2}{3} \times \frac{3}{4}$ .



$14 \times 8 = 112$   
Use this to work out:

$$\begin{array}{l} 1.4 \times 8 \\ 140 \times 8 \end{array}$$

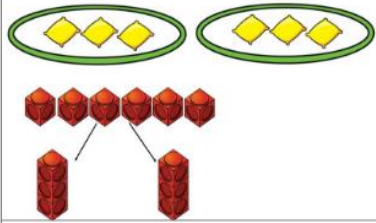
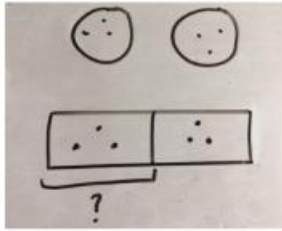
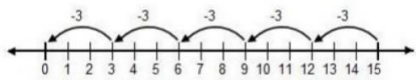
$$\begin{array}{r} 1\ 2\ 4 \\ \times\ 2\ 6 \\ \hline \phantom{1}7\ 4\ 4 \\ \phantom{1}2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Work out the missing number.

$$6 \times 35 = \square \times 5$$

			<div style="text-align: center; margin-bottom: 10px;"> <span style="border: 1px solid black; padding: 2px 5px;">2</span> <span style="border: 1px solid black; padding: 2px 5px;">3</span> <span style="border: 1px solid black; padding: 2px 5px;">4</span> <span style="border: 1px solid black; padding: 2px 5px;">5</span> <span style="border: 1px solid black; padding: 2px 5px;">7</span> <span style="border: 1px solid black; padding: 2px 5px;">8</span> </div> <p style="text-align: center; font-size: small;">Place the digits in the boxes to make the largest product.</p> <div style="text-align: center; margin: 10px 0;"> <table style="margin: auto; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> </tr> <tr> <td style="border: none; width: 10px;"></td> <td style="border: none; width: 10px;"></td> <td style="border: none; width: 10px;"></td> <td style="border: none; width: 10px;"></td> </tr> <tr> <td style="border: none; width: 10px;"></td> <td style="border: none; width: 10px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> </tr> <tr> <td colspan="4" style="border-top: 1px solid black; border-bottom: 1px solid black;"></td> </tr> </table> </div> <p style="text-align: center; font-size: small;">Calculate</p> <div style="display: flex; justify-content: space-around; font-size: small;"> <div style="text-align: center;"> <math display="block">\begin{array}{r} 4267 \\ \times 34 \\ \hline \end{array}</math> </div> <div style="text-align: center;"> <math display="block">\begin{array}{r} 3046 \\ \times 73 \\ \hline \end{array}</math> </div> </div>																

<b>Division</b> <b>Year 1</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>		
<p>Pupils should be taught to: -step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p> <p><b>Concrete:</b> counting sticks, 100 squares and numberlines, Numicon, arrays through Numicon grouping and sorting</p> <p><b>Pictorial:</b> using arrays, grouping and sorting</p>	<p style="font-size: x-small;">Sharing using a range of objects. 6 ÷ 2</p>  <p>The diagram shows 6 yellow diamonds grouped into two groups of 3. Below, 6 red cubes are shown being divided into two groups of 3 cubes each.</p>	<p style="font-size: x-small;">Represent the sharing pictorially.</p>  <p>The diagram shows two faces, each with 3 dots. Below, two boxes each containing 3 dots are shown, with a bracket underneath and a question mark, representing the division problem.</p>	<p style="font-size: x-small;">6 ÷ 2 = 3</p> <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 40px; height: 20px;">3</td> <td style="width: 40px; height: 20px;">3</td> </tr> </table> <p style="font-size: x-small;">Children should also be encouraged to use their 2 times tables facts.</p>  <p>The numberline shows a sequence from 0 to 15. Five arrows point left from 6, 3, 0, 3, and 6, each labeled '-3', illustrating the repeated subtraction process.</p>	3	3
3	3				

**Abstract:** using numberlines, linking division to repeated subtraction, working out the inverse, grouping and sorting

**Year 2**

Pupils should be taught to: division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs numbers can be done in any order (commutative) and division of one number by another cannot multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

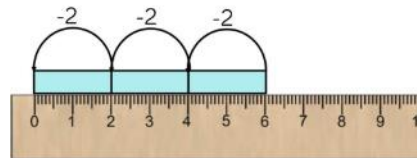
**Concrete:** counting sticks, 100 squares and numberlines, Numicon, arrays through Numicon grouping and sorting

**Pictorial:** using arrays, grouping and sorting

**Abstract:** using numberlines, linking



Repeated subtraction using Cuisenaire rods above a ruler.  $6 \div 2$



3 groups of 2

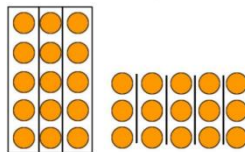
$2d + 1d$  with remainders using lollipop sticks. C rods, above a ruler can also be used.  $13 \div 4$

Use of lollipop sticks to form wholes- squares because we are dividing by 4.



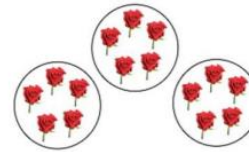
There are 3 whole squares, with 1 left over.

To solve problems such as  $15 \div 3 =$ , children will share 15 objects into 3 groups like in the first array or make groups of 3 until they get to 15, like in the second image.



Completing both of these processes will help children see the link between sharing and grouping but also the link between  $15 \div 3 = 5$  and  $15 \div 5 = 3$ .

A farmer has 15 roses and shares them between 3 friends. How many roses do they each get?



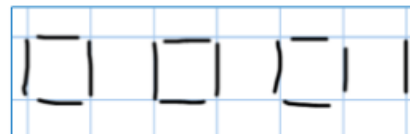
15 roses shared between 3 = 5 roses each

Bats fly in groups of 2. How many groups of 2 will there be if there are 8 bats?



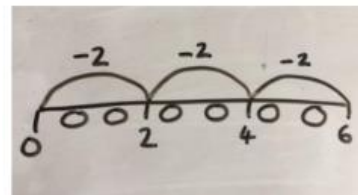
8 bats shared into groups of 2 = 2 bats in each group

Children to represent the lollipop sticks pictorially.



There are 3 whole squares, with 1 left over.

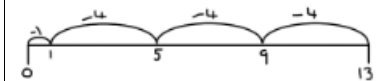
Children to represent repeated subtraction pictorially.



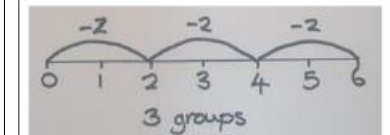
$13 \div 4 = 3$  remainder 1

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'



Abstract number line to represent the equal groups that have been subtracted.



division to repeated subtraction, working out the inverse, grouping and sorting

### Year 3

Children should:

recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

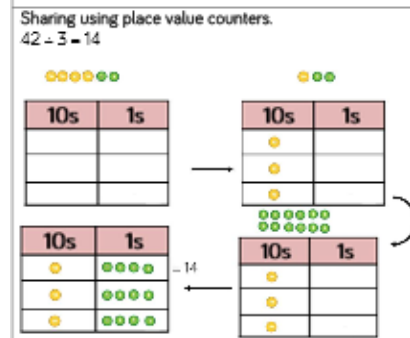
solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects

#### Concrete methods:

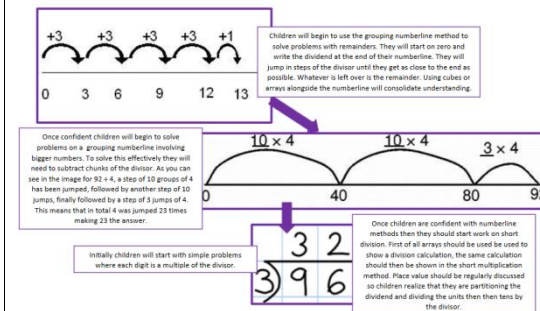
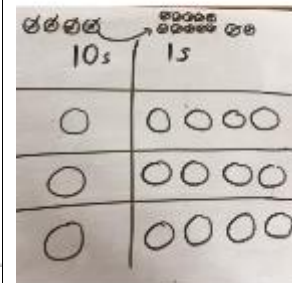
linking multiplication to repeated addition, using cones, Numicon, multilink or Cuisenaire rods to look at arrays with related multiplication and division facts, using place value charts to multiply tens and units, using Numicon to group tens and units

#### Pictorial methods:

Linking Numicon to more formal methods for multiplication such as the grid method, using shape to complete number statements, showing arrays pictorially, showing Dienes pictorially by



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$42 \div 3 = 14$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

$$10 + 4 = 14$$

multiplying out tens and units

**Abstract:**

Completing bar models, using jottings to support concrete or pictorial methods by multiplying tens and units, starting to use formal methods for multiplication, linking multiplication to measure and problem solving

**Year 4**

Pupils should:

recall multiplication and division facts for multiplication tables up to  $12 \times 12$   
 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

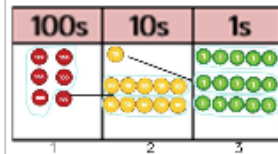
recognise and use factor pairs and commutativity in mental calculations  
 multiply two - digit and three - digit numbers by a one - digit number using formal written layout

solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

**Concrete methods:**

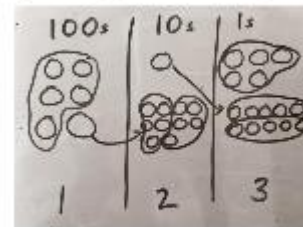
linking division to repeated subtraction, using cones, Numicon, multilink or Cuisenaire rods to look at arrays with related multiplication and division facts,

Short division using place value counters to group.  
 $615 \div 5$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

$$\begin{array}{r} 12 \\ 8 \overline{) 96} \end{array}$$

Once confident with the method of short division, they will move on to problems where the first digit of the dividend is not a multiple of the divisor and therefore a remainder will need to be carried. Children may need to use other equipment to calculate the division and multiplication facts required.

Children who can use short multiplication problems with remainders that not those in the final answer) are now ready to work on 3 digit problems. Again, there should be remainders in the calculation but none in the final answer.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$$

$$\begin{array}{r} 035 \\ 5 \overline{) 175} \end{array}$$

Once children are confident at dividing with 3 digits, they need to attempt problems where the answer in the first column (hundreds column) is a zero. They may wish to record the hundred initially as this will help them remember its place and the numbers value.



using place value charts to divide tens and units, using Numicon to group tens and units

**Pictorial methods:**

Linking Numicon to more formal methods for division such as the short division, showing arrays pictorially, showing Dienes pictorially by dividing out tens and units

**Abstract:**

Completing bar models, using jottings to support concrete or pictorial methods by divide tens and units, starting to use formal methods for division, linking division to measure and problem solving such as finding missing sides if you know the area, converting units of measure and being able to divide by 10 and 100

**Year 5**

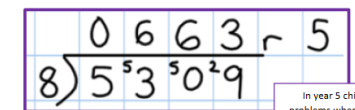
multiply and divide numbers mentally drawing upon known facts

the efficient written method of short division and interpret remainders appropriately for the context

multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

solve problems involving multiplication and division, including scaling by simple



In year 5 children will begin to solve problems where a number up to 4 digits is divided by a single digit number including an remainder. These division problems are contextual so the children learn how to interpret a remainder - as a number, a fraction, rounded up or rounded down

Use the division diagram to help solve the calculations.  
 $7,200 \div 200 = 36$

7,200	+	100	$3,600 \div 200 = \square$
72	+	2	$18,000 \div 200 = \square$
36	+	2	$5,400 \div \square = 27$
			$\square = 6,600 \div 200$

fractions and problems involving simple rates

**Concrete** - using counters for division to support more formal methods

**Pictorial** - using jottings as a way to move on to more formal methods for division

**Abstract** - using short division methods, using known facts to solve problems, completing bar models, complete missing numbers questions

### Year 6

divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to context

use written division methods in cases where the answer has up to two decimal places

use common factors to simplify fractions; use common multiples to express fractions in the same denomination

divide proper fractions by whole numbers (e.g.  $\frac{1}{3} \div 2 = \frac{1}{6}$ )

associate a fraction with division to

Long division using place value counters  
2544 ÷ 12

We can't group 2 thousands into groups of 12 so we'll exchange them.

We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 0.2 \\ 12 \overline{) 2544} \\ \underline{24} \phantom{00} \\ 14 \phantom{00} \\ \underline{12} \phantom{00} \\ 24 \phantom{00} \\ \underline{24} \\ 0 \end{array}$$

After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

After exchanging the 2 tens, we have 14 ones. We can group 14 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 0.212 \\ 12 \overline{) 2544} \\ \underline{24} \phantom{00} \\ 14 \phantom{00} \\ \underline{12} \phantom{00} \\ 24 \phantom{00} \\ \underline{24} \\ 0 \end{array}$$

Dividing Fractions By Whole Numbers

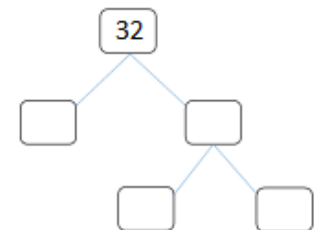
$$\frac{2}{3} \div 2 = \frac{2}{6}$$

the number of shaded boxes in one circle

total number of boxes

What are the common factors of these pairs of numbers?

24 and 36  
20 and 30  
28 and 45



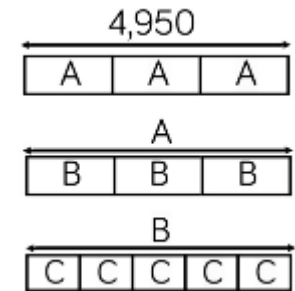
calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. 3/8)

identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places

**Concrete** - using counters for division to support more formal methods such as short and long division

**Pictorial** - using jottings as a way to move on to more formal methods for division

**Abstract** - using short and long division methods, using known facts to solve problems, completing bar models, complete missing numbers questions, using images to support dividing fractions by whole numbers



$$\begin{array}{r} 041\text{r}3 \\ 4 \overline{)159} \end{array}$$

$$\begin{array}{r} 12 \\ 432 \\ - 36 \\ \hline 72 \\ - 72 \\ \hline 0 \end{array}$$

Use the long division method to solve the following calculations. One has been done for you as an example.

$$\begin{array}{l} 836 - 11 = \\ 798 - 14 = \\ 608 - 19 = \end{array}$$

$$\begin{array}{r} 0812.125 \\ 8 \overline{)6497.000} \end{array}$$

The remainder in this answer would have been 3 but it has been expressed as a decimal. To do this, children need to insert a decimal point next to the units and carry the remainder over the decimal point. Zeros are inserted to the right of the decimal point to show that there was no value.

The focus in year 6 is not so much the method of short division but how the remainders are expressed- children need to express remainders as decimals and fractions depending on the context of the question.

A great way of remembering the steps of long division is *Goes McDonalds Give Burgers?*



$$\begin{array}{r} 291 \\ 45 \overline{)13095} \\ \underline{90} \\ 409 \\ \underline{405} \\ 45 \end{array}$$

To divide by 2 digit numbers, the children will use the method of long division. The example to the right clearly shows the method in the 'Burger' steps, where as the example to the left shows what a completed method would look like. Any remainders would need to be expressed in a way that matched the context of the problem.